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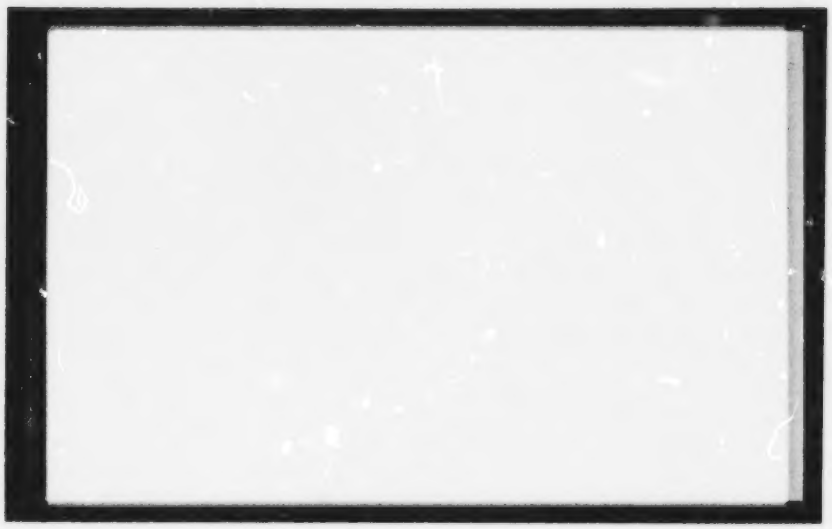

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A Model for Large Scale Aircraft Routing and Scheduling Problems+

by

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Abstract

The model described determines airline flight schedules which will maximise company profits and passenger satisfaction. The approach represents a significant improvement in analytical methods for fleet planning and scheduling which take into account the complex interaction between the passenger and aircraft routing components. In spite of the high level of detail in the description of these components, the solution time is kept low by the use of powerful mathematical programming techniques. Moreover, the model can easily be adapted to deal with several different formulations.

1. INTRODUCTION

Given the estimated demand for travel, an airline wishes to determine the flight schedule which maximizes its profit while taking into account the satisfaction of its customers. In this system, two components interact: the aircraft routing in the physical network determines a flight schedule, and the passengers use these flights to travel. The solution technique proposed is a heuristic adaptation of the Frank-Wolfe algorithm for an integer programming problem with a special structure. It corresponds to solving alternately the aircraft routing problem to select the flights that improve profits, and the passenger assignment problem to these flights to estimate revenues.

The model described in this paper could be used as a planning tool by an airline. The decision variables are the fleet, the frequency plan and the schedule. The degree of control over these variables depends on the planning horizon. Thus, in a long-range planning model it is assumed that the fleet may vary, while in a mid-range planning model, the fleet is usually fixed but the frequency plan may be modified.

In Section 2, the physical network of aircraft routes is introduced together with a general model which corresponds to the latest thinking on the problem. Certain modifications are proposed which result is a more realistic model which is also easier to handle. The passenger assignment subproblem is analysed in Section 3 where the paper of Soumis et al [18] is summarized. In Section 4, the aircraft routing problem is analysed. The adaptation of the Frank-Wolfe algorithm is presented together with the technique for the transfer of information from the passenger problem to the aircraft routing problem. In Section 5, the algorithm is presented.

2. MODEL DESCRIPTION

In this section a mathematical model corresponding to the most recent thinking on the subject is described together with some of its variants. The proposed approach is then outlined. Before the models are introduced, some basic definitions are presented and the physical network for aircraft is defined.

2.1 Basic Definitions

A flight, is defined by its itinerary (Montreal-Chicago-Los Angeles), its departure and arrival times, and the aircraft type used. A flight-leg is a non-stop component of a flight (Montreal-Chicago). A route is composed of flight-legs not necessarily belonging to the same flight. For instance, a route from Montreal to San Francisco may be composed of the flight-leg Montreal - Chicago (from the flight Montreal - Chicago - Los Angeles) and of the flight-leg Chicago - San Francisco (from the flight Toronto - Chicago - San Francisco).

Finally note that these models analyse the operations of the airline on an average day, and thus the number of passengers assigned to each route by the model is in fact an average number of passengers desiring to use the route. It is also assumed that the airline can specify a set of candidate flights probably including most of those in operation.

Associated with each aircraft type there is a network specified as follows. Consider a space-time network as illustrated in Figure 1 where there is a node for each city each time a candidate flight starts or ends there, and where each arc corresponds to either a candidate flight, or a return arc, or a link between consecutive time periods. Note that a return arc makes the aircraft in a given city at the end of the day available in the same city in the morning of the next day.

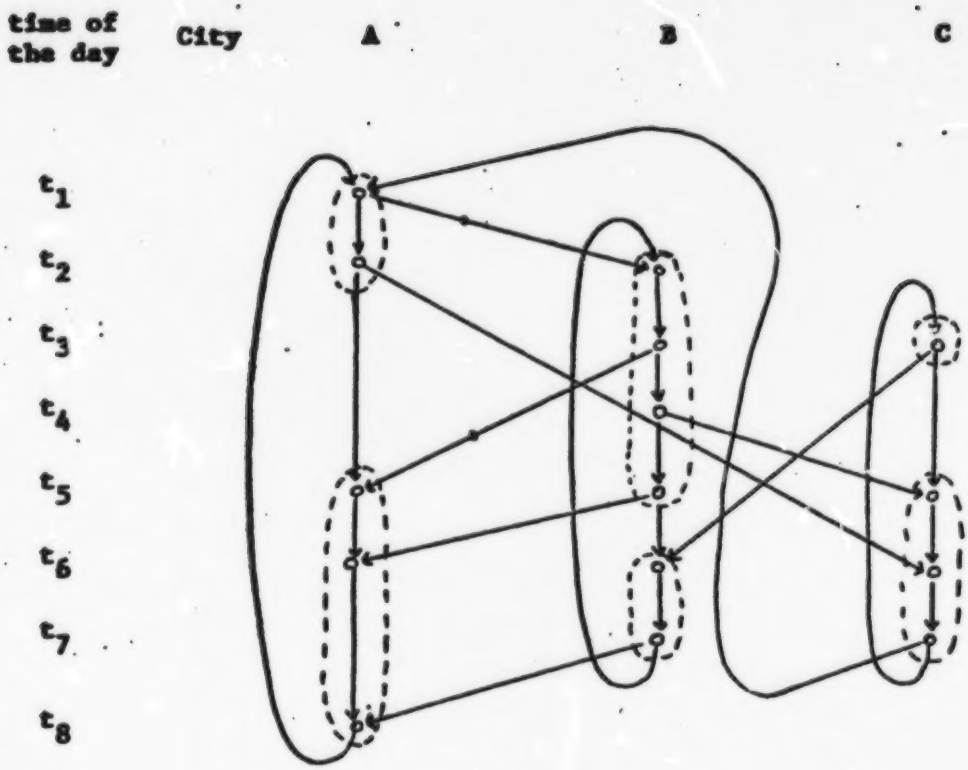

FIGURE 1 : SPACE-TIME AIRCRAFT NETWORK

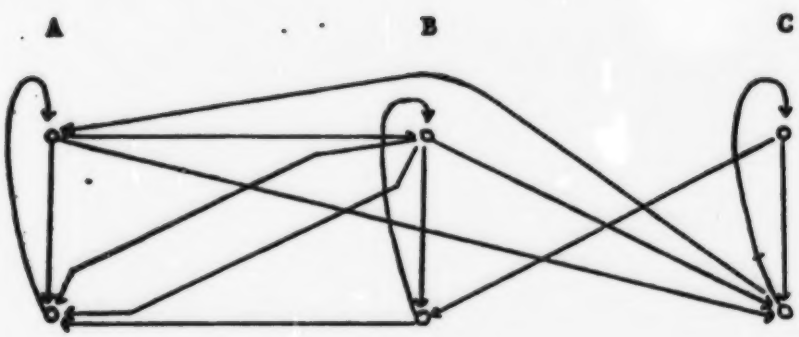

FIGURE 2 : AGREGATE AIRCRAFT NETWORK

The network illustrated in Figure 2 is constructed from the space-time network by aggregating nodes. The aggregation rule ensures that a flight uses an aircraft which will be available before its departure time.

All the arcs corresponding to flights have a capacity of one and all the return arcs and the arcs linking the node in time at each city (i.e. vertical arcs in Figure 2) do not have any capacity restriction.

The cost structure is defined as follows. The operating cost for each flight is associated with the corresponding arc. To integrate into the model the investment cost for the purchase of an aircraft, a reference time (midnight for instance) is used when the aircrafts of a given type are counted. An amortized investment cost is added to all the arcs (link, return or flight arcs) that cross this point in time. It should be noted that this cost may be specified by the airline to integrate other indirect costs and planning considerations.

For example, the Air Canada network with 250 actual flights and 250 candidate flights generates about 500 linking and return arcs for a problem of about 1000 arcs and 500 nodes.

It is also useful to introduce here the following notation.

Passenger Component

Δ is the set of markets (a market being specified by an origin-destination pair and an interval of time during the day).

N_m is the number of passengers in market m.

Q_m is the set of routes available for market m.

h_r^m is the number of passengers from market m using route r.

h (h_r^m), $r \in Q_m$, $m \in \Delta$, is a passenger assignment.

B^m is the ticket price for market m.

τ_r^m a monetary equivalent for the dissatisfaction of the passengers of a market m travelling on route r, $\tau_r^m > 0$

δ_ℓ^r – $\begin{cases} 1 & \text{if route r uses flight-leg } \ell \\ 0 & \text{otherwise} \end{cases}$

Aircraft Component

L is the set of flight-legs

J is the set of indices of the arcs corresponding to candidate flights

J_K is the set of indices of the arcs corresponding to candidate flights using aircraft type K.

N is the set of arcs in the aircraft network

y_j is the number of aircraft on arc $j \in N$ (y_j is a binary variable for $j \in J$)

γ_ℓ^j – $\begin{cases} 1 \text{ if flight j includes flight-leg } \ell \\ 0 \text{ otherwise} \end{cases}$

r_j is the number of seats available on flight j

y (y_j), $j \in N$, is an aircraft routing

g_j is the cost on arc j

$g(y)$– $\sum_{j \in N} g_j y_j$ is the cost of the routing y

2.2 Fleet planning models in the literature

Several linear programming models have been used for the fleet planning problem. The most complete formulations were proposed by Agin [2], Chan [5], Richardson [16] and Scarbrough [17] who considered the aircraft routing problem and its interaction with the passenger assignment problem. A typical mathematical formulation for these models is given in Figure 3. The objective is in general to minimize the difference between the utilization cost of the aircraft and passenger revenue, taking into account the loss of revenue associated with passenger dissatisfaction with the quality of service. Constraints of the type 2.1 ensure that the demand from a market is distributed among the available routes serving this market. Constraints 2.2 imply that the average daily number of passengers must not exceed the capacity of the aircraft times a predetermined load factor. Equations 2.3 are flow conservation constraints for each type of aircraft.

The block structure of the problem suggests the use of the Dantzig-Wolfe decomposition. Hence constraints (2.1) and (2.2) correspond to the Master and each block in (2.3) generates a sub-problem. Unfortunately, this approach generates a Master problem that is a large multicommodity flow problem (each market corresponding to a commodity) with explicit bounds on the total flow on each flight-leg. This type of problem is very expensive to solve and this approach has rarely been used. Agin [2] did not test his method. Richardson [16] and Scarbrough [17] analyse only small problems because of computer time and memory limitations. Finally, Chan [5] proposed more efficient network flow techniques for large scale problems but his algorithm was implemented only in limited situations where for example the freedom to use flights with stops and/or connections is restricted and only one type of aircraft is available.

TYPICAL FLEET-PLANNING MODEL IN THE LITERATURE

$$\text{Min} \left\{ g(y) - \sum_{m \in \Delta} \sum_{r \in Q_m} (B^m - \tau_r^m) h_r^m \right\}$$

(2.1) $\sum_{r \in Q_m} h_r^m = N_m, \quad m \in \Delta$

(2.2) $\sum_{m \in \Delta} \delta_\ell^r \, h_r^m - \sum_{k=1}^{K} \sum_{j \in J_k} \Gamma_j \gamma_\ell^j y_j \leq 0, \quad \ell \in L$

(2.3) Flow conservation

Type 1

. . .

Type K

(2.4) $h_r^m \geq 0 \quad m \in \Delta, \; r \in Q_m$ | $y_j = 0$ or $1 \quad j \in J$

PROPOSED MODEL

$$\text{Min} \left\{ g(y) + p(h,y) \right\}$$

$\sum_{r \in Q_m} h_r^m = N_m, m \in \Delta$ — Satisfaction of passenger requirements

Aircraft Capacity Constraints

Flow conservation

Type 1

. . .

Type K

Aircrafts circulation

$h_r^m \geq 0 \quad m \in \Delta, \; r \in Q_m$ | $y_j = 0$ or $1 \quad j \in J$

Figure 3 AIRLINE PLANNING MODELS

Other more easily solvable variants of this flight planning model have been proposed. Swan [21] describes such a model developed at the Flight Transportation Laboratory. Equations 2.3 are approximated by constraints bounding the average utilization of each aircraft type by a predetermined number corresponding generally to the observed utilization. In addition, the demand is a function of the quality of service. Thus the dissatisfaction term in the objective function is dropped. Agard et al [1] also use such an approximation and in addition, drop the h variables and the constraints 2.1 assuming that the average number of passengers on each flight-leg is known. Price et al [15] however use a model defined by equations 2.1 and 2.2 for a middle sized passenger routing problem. These simpler variants have been used on a certain number of airline networks. Their usefulness for decision making is however limited because of the approximations used.

2.3 Proposed model

The modified version proposed eliminates the capacity constraints (2.2) at the expense of a more complex objective function. This function now includes a term corresponding to each flight-leg. This term is an increasing function of the number of passengers assigned to the flight-leg. This function represents the loss of revenue due to exceeding the capacity of the aircraft. This modification leads to a more realistic approach to modelling the demand variability from day to day and furthermore produces a problem which is much easier to solve.

With this modification, the problem is formulated in Figure 3 as an aircraft routing and scheduling problem and summarized as follows.

2.5 $$\min_{y \in Y} \{g(y) + \min_{h \in H(y)} p(h,y)\}$$

where

$Y = \{y \mid y$ is an aircraft routing satisfying constraints (2,3)$\}$

$H(y) = \{h \mid h_r^m > 0,\ m \in \Delta,\ r \in Q_m$, satisfying constraints (2,1)$\}$

$p(h,y)$ is the loss of revenue due to passenger dissatisfaction and lack of space on the aircraft when routing y is operated.

Note that the function in y to be minimized is not in a closed form since a minimization of the passenger assignment problem is required. Thus the Frank-Wolfe approach will be adapted to take this into account and to generate a technique for moving from one feasible solution to another in order ot reduce the overall cost.

Before we move to the section on the solution technique, it is necessary to summarize a model for the passenger assignment problem described in Soumis et al [18.]

3. PASSENGER ASSIGNMENT MODEL

A passenger assignment model must forecast how passengers use a given flight schedule to travel from their places of origin to their destinations. It should be noted, however, that the model described here works on the basis of the "system optimization" approach, as do most other models in the literature. This assumes that the passengers behave in such way that the global objective is maximized. This approach gives good results in spite of its simplicity and its theoretical weakness compared with the more realistic "user optimization" approach where each passenger seeks to maximize his own satisfaction.

Before proceeding further, the notion of a market must be developed in more detail. Recall that a market corresponds to the demand for an origin-destination pair during a given period of the day. This is illustrated in Figure 4.

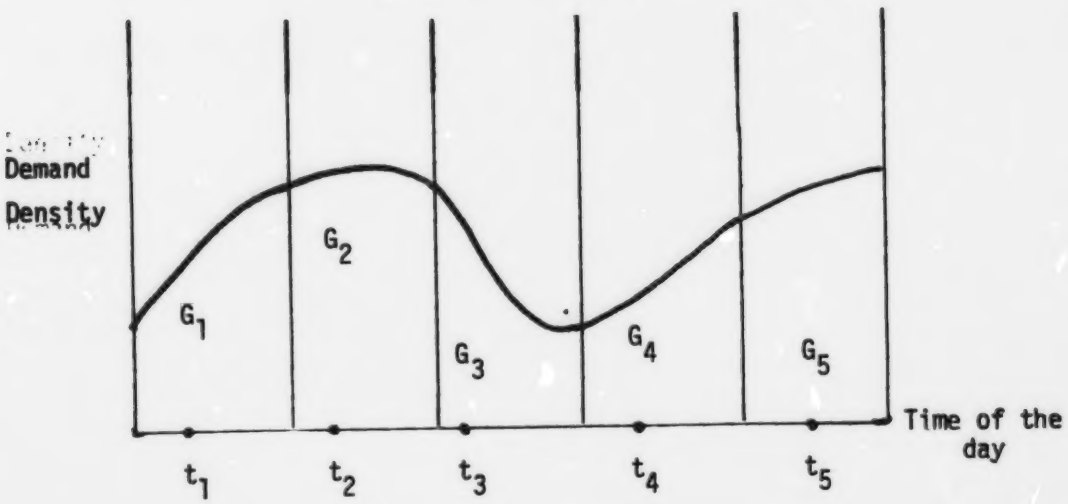

FIGURE 4 : DEFINITION OF MARKETS

$G_i (i = 1,2,\ldots,5)$ correspond to markets, and all the passengers in group G_i are assumed to have the average desired departure time t_i. In our application, for a given origin-destination pair, there are as many markets as there are routes serving this city-pair. Thus, the markets are defined differently according to the flight schedule in operation, (Note that this explains the notation H(y) in (2.5).) Finally, in our application, the passengers in a given market G_i are restricted to using one of the three most appropriate routes relative to the desired departure time t_i in order to reduce the number of variables h_r^m. This restriction did not alter the quality of the results obtained. Hence the three variables

$h^i_{r_1}$, $h^i_{r_2}$, and $h^i_{r_3}$ associated with G_i take values corresponding to the number of passengers in G_i using routes r_1, r_2, and r_3, respectively.

The objective function p(h,y) is composed of two factors, one related to the dissatisfaction associated with the routes and the other with the capacity constraints of the aircraft.

For our requirements, a fairly standard approach (Boile [4], Eriksen and Teneja [8], [9]) is used for the route dissatisfaction factor. The factor τ is linear; it takes into account the difference between the desired departure time and the flight departure time, and the increase in travel time due to connections and/or stops. Other formulations could be used.

To integrate the capacity constraints into the objective function, we introduce the concept of "passengers in excess" to take into account the variation in the daily demand. Notice that this concept corresponds to the notion of "spill" recently introduced in Lankford and Pina [14] to analyse and forecast the surplus of seats to be offered at discount fares.

The approach is to determine for each flight-leg a non-linear function giving the average number of passengers in excess (or refused) in terms of the average number of passengers assigned to the flight-leg by the model. Hence in general, not all the passengers assigned to a flight will travel. Thus the capacity constraints (2.2) become implicit, and the interaction between the passengers of different markets sharing flight-legs is specified through these non-linear functions.

In order to transform the information at flight-leg level into information for the individual markets, the following hypotheses are introduced:

Hypothesis 1 : The probability that a passenger assigned to a flight-leg is in excess is independent of the origin-destination pair to which he belongs.

Hypothesis 2 : For an optimal assignment of passengers, the probability that the same passenger is in excess on more than one flight-leg making up the route is equal to zero.

These hypotheses are discussed in Soumis et al [18] and have been found to be quite realistic.

Under these hypotheses the passenger problem can be formulated as follows:

$$\text{Min } \{ \sum_{m\in\Delta} \sum_{r\in Q_m} B^m h_r^m \sum_{a\in F_r} \frac{Z_a(f_a)}{f_a} \} + \{ \sum_{m\in\Delta} \sum_{r\in Q_m} \tau_r^m h_r^m \}$$

$$\text{Subject to } \sum_{r\in Q_m} h_r^m = N_m \quad , \ m \in \Delta$$

$$h_r^m \geq 0 \quad , \ m \in \Delta, \ r \in Q_m,$$

where $f_a = \sum_{m\in\Delta} \sum_{r\in E_a^m} h_r^m$ is the number of passengers assigned to flight-leg a,

$Z_a(f_a)$ is the average number of passengers in excess for flight-leg a,

F_r is the set of flight-legs making up route r,

E_a^m is the set of routes serving the market m that use flight-leg a.

Problems with such a block-structure of the constraints and a convex objective function can be solved by a straight-forward decomposition technique. Each subproblem corresponding to a block (market) is optimized keeping the value of the variables of the other blocks fixed. The solution is optimal when we run through all the subproblems without improving the value of the objective function. Each subproblem has a non-linear objective to be minimized and only one linear constraint. These subproblems have been solved using an iterative technique where the improving direction is determined through a second order approximation of the objective function (Ferland et al [10]). Such a decomposition technique has been used even when the objective function is not convex.

Finally, note that this is a very efficient technique since a problem of 6000 variables and 2000 constraints corresponding to the complete Air Canada System is solved in less than 20 seconds C.P.U. (C.D.C. 173). Using the observed demand for travel for a three month period, the average absolute error between the passenger routing obtained from the model and the actual passenger routing observed was less than 11 passengers per flight-leg. These results are considered quite acceptable by Air Canada and the industry in general. It is not expected that much better results could be easily attained. A detailed analysis of the results obtained with our model is presented in Soumis et al [18].

4. AIRCRAFT ROUTING AND SCHEDULING PROBLEM

As mentioned in section 2.3, the aircraft routing and scheduling problem (2.5)

$$\underset{y \in Y}{\text{Min}} \{g(y) + \underset{h \in H(y)}{\text{Min}} p(h,y)\}$$

is to be solved with a Frank-Wolfe type technique. For the sake of completeness the standard Frank-Wolfe technique is summarized, and then its adaptation to solve (2.5) is described.

4.1 Standard Frank-Wolfe Technique

The Frank-Wolfe method to solve the problem

$$\underset{y \in Y}{\text{Min}}\ C(y)$$

(C being a differentiable function of Y) is an iterative technique moving from one feasible solution y^k to another y^{k+1} where $C(y^{k+1}) > C(y^k)$. Furthermore y^{k+1} is determined form y^k by the relation:

$$y^{k+1} = y^k + \sigma_k\, d^k$$

where d^k is a descent direction and σ_k is the optimal step size in this direction. More specifically, d^k is the steepest descent direction defined as an optimal solution to the problem

$$\text{Min } (\nabla C(y^k),\ y^k + d)$$

(4.1)

$$\text{Subject to } (y^k + d) \in Y.$$

The optimal step size is determined by minimizing the objective function in the direction d^k:

$$C(y^k + \sigma^k d^k) = \underset{0 \le \sigma \le 1}{\text{Min}} \{C(y^k + \sigma\, d^k)\}.$$

4.2 Adapted Technique

Two adaptations of the standard technique are necessary to take into account the structure of the problem. In the first place, fractional values for the variables y_i are not acceptable since these variables correspond to numbers of aircraft. Secondly, a way must be found to linearize the objective.

$$g(y) + \min_{h \in H(y)} p(h,y)$$

which is not in closed form.

Recall that the feasible set Y corresponds to constraints of a network circulation problem separable by aircraft type. It is well known that a network problem with a linear objective function has an optimal solution with integer valued variables.

Since the successive objective functions of the Frank-Wolfe algorithm are linear approximations of the original objective, it suffices to accept only step size σ^k egal to 0 or 1 to ensure integrality of solutions throughout the algorithm: i.e.; if

$$g(y^k + d^k) + \min_{h \in H(y^k + d^k)} p(h, y_k^k + d^k) < g(y^k) + \min_{h \in H(y^k)} p(h, y^k),$$

$$\text{then take} \quad y^{k+1} = y^k + d^k,$$

otherwise try to determine another direction. It is worth noting the similarity between this approach and the "reflection programming" technique in Agin [2], [3]. This iterative technique moves only from one aircraft routing to a better alternate routing. Unfortunately, this is a suboptimal technique since it can generate local minima, but it has the advantage of preserving integrality.

The only thing to be specified in order to completely describe the technique is the linearization of the objective

$$g(y) + \min_{h \in H(y)} p(h,y)$$

at each iteration. Recall that in the standard technique this linearization is achieved by using the gradient as the vector of coefficients for the variables.

In other words, at the k^{th} iteration we have to estimate

$$\frac{\partial}{\partial y_j}\left(g(y) + \underset{h\in H(y)}{\text{Min}}\ p(h,y)\right)\Big|_{y=y^k} = g_j + \frac{\partial}{\partial y_j}\left(\underset{h\in H(y)}{\text{Min}}\ p(h,y)\right)\Big|_{y=y^k}$$

$$= g_j + \bar{c}_j^k$$

For the arc $j \in N-J$ (i.e. for return arcs)

$$\bar{c}_j^k = \frac{\partial}{\partial y_j}\left(\underset{h\in H(y)}{\text{Min}}\ p(h,y)\right) = 0$$

since no passengers are assigned to these arcs. For arcs $j \in J$ (i.e. for arcs corresponding to flights), since aircraft routing modifications correspond to adding and removing flights (σ^k, the step size, being equal to 0 or 1), it follows that it is preferable to estimate the finite difference

$$\pi_j^k = \underset{h\in H(y^k + \Delta_j)}{\text{Min}}\ p(h,y^k + \Delta_j) - \underset{h\in H(y^k)}{\text{Min}}\ p(h,y^k) \tag{4.2}$$

where Δ_j is equal to the unit vector e_j ($-e_j$) if the inactive (active) flight j in y^k is added (removed). Figure 5 illustrates the advantage of the approach using π_j's instead of $\bar{c}_j$'s.

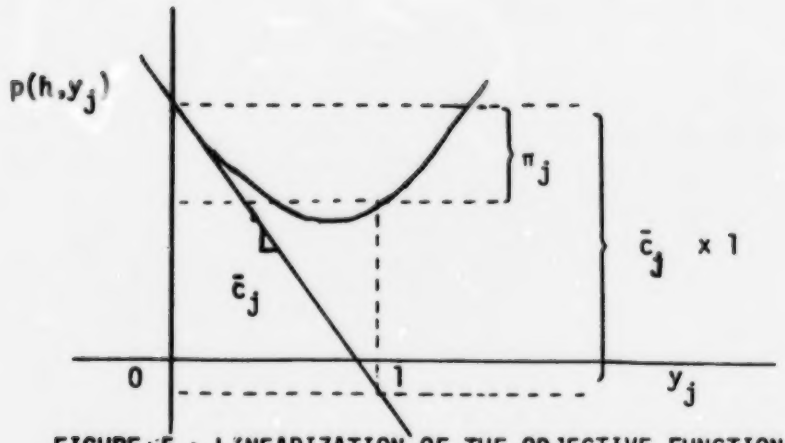

FIGURE 5 : LINEARIZATION OF THE OBJECTIVE FUNCTION

Thus, the descent direction d^k used in the adapted Frank-Wolfe technique is defined as an optimal solution to the problem

$$\min \sum_{j \in N} (g_j + \pi_j^k)(y_j^k + d_j)$$

Subject to $(y^k + d) \in Y$

Where d_j is the j^{th} component of d.

To evaluate π_j^k a passenger assignment problem must be solved with the flight schedule $y^k + \Delta_j$. To reduce the computation time, approximate values of the π_j^k are used instead. These values are obtained as follows.

If a flight is to be removed from the schedule, then some routes for some markets are not available anymore. Only the passengers that were on the routes removed are assigned to alternate routes and π_j^k is taken to be equal to the corresponding increase in the total cost. This is a conservative estimate since we slightly overestimate the cost increase.

If a new flight is to be added to the schedule, then new routes are created for some markets. For the origin-destination pairs not concerned by this addition the passenger assignment remains unchanged, and the assignment for the other pairs are reoptimized. π_j^k is taken to be equal to the negative value of the decrease in cost. This is also a conservative estimate.

With these values for the π_j^k we are neglecting the cross influence of introducing and/or removing several flights, and thus it is not necessarily true that a good decreasing direction d^k for the linear approximation $\sum_{j \in N} (\pi_j^k + g_j)(y_j^k + d_j^k)$ introducing many modifications to y^k implies that

$$g(y + d^k) + \underset{h \in H(y + d^k)}{\text{Min}} p(h, y + d^k) < g(y) + \underset{h \in H(y)}{\text{Min}} p(h, y)$$

Furthermore, the values of the π_j^k depend heavily on the aircraft routing y^k. Thus, at least during the first iterations, we are not seeking for an optimal solution of (4.3) but rather for a slight modification of the routing to reduce the overall cost.

5. THE COMPLETE ALGORITHM

The algorithm is summarized as follows

Step 0

Determine an initial solution. For instance, the actual flight schedule and routing of the airline may be used to determine the passengers flows.

Step 1

Determine the value of π_j^k for each flight.

Step 2

Determine an alternate aircraft routing by solving (not necessarily to optimality)problem (4.3).

Step 3

Determine the passenger flows corresponding to the proposed routing and evaluate the overall cost (cost for passengers in excess, plus cost due to passenger dissatisfaction, plus aircraft cost). If the cost has decreased, then retain the routing and repeat step 1 with it.

Step 4

If this proposed routing is an optimal solution for problem (4.3), then terminate the algorithm. Otherwise determine a better alternate aircraft routing by continuing to solve problem (4.3). Then repeat step 3 for this routing.

Several solution strategies should be analysed. For instance in Step 1, instead of estimating the π_j^k for each flight, only those for the flights using some specific aircraft types might be determined and then in Step 2 an alternate routing found by solving the subproblems associated with these aircraft only. Also, in Step 2, we have to specify a criterion to decide when to move to Step 3. Only numerical experimentation will indicate the best strategies to use at the different steps of the algorithm.

The solution techniques used to execute the steps of this algorithm should be very efficient. Our estimates of the solution times are given in table 1.

TIME IN SECONDS (CPU CDC 173)

INITIALIZATION	
- Network set up	30
- Passenger assignment	20
- Initialization time	50
SUBSEQUENT ITERATIONS	
- Step 1 : calculation of π_j^k	5
- Step 2 : aircraft routing improvement	3
- Passenger network update	10
- Step 3 : reoptimization of passenger assignment	8
ITERATION TIME	26

TABLE 1 : TIME ESTIMATES

These initialization times were taken directly from MAPUM [18]. The time associated with the calculation of the π_j^k and the reoptimization of the passenger assignment (Step 1 and Step 3) were extrapolated from our experience with MAPUM. For networks of this size, our experience is that more than 100 flow modifications can be performed in less than 3 seconds. The time estimate for the aircraft routing improvement should be even less because fewer modifications will be made at each iteration. The time associated with the passenger network update was estimated at one third of the setup time. Hence it is expected that approximately 20 iterations could be performed in 10 minutes of computer time.

6. CONCLUSION

This model represents a step forward in analytical methods for the fleet planning and scheduling problem which take into account the complex interaction between the passenger component and the aircraft component. The notion of "passengers in excess" allows a more realistic treatment of this interaction. Furthermore, although these two components are treated at a high level of detail, the solution time is kept low by the use of decomposition principles and other powerful mathematical programming techniques.

Moreover, this framework of analysis is easily adaptable. For instance, the approach used here to determine passenger assignments to flights is of the "system optimization" type as opposed to the "user optimization" approach mentioned in section 3. A similar solution technique could be used to determine an optimal assignment with the latter approach. The model could also be modified to integrate the notion of variable demand. An analysis of these modifications and extensions has been made by Soumis [19] but the data needed to test these methods are not available.

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